For all questions, answer choice "E) NOTA" means that none of the above answers is correct.

1. How many distinct permutations are there for the letters in the word MUALPHATHETA?

A) $\frac{12!}{4!}$ B) $\frac{12!}{3!}$ C) $\frac{12!}{2!}$ D) $12!$ E) NOTA

2. A fair standard six-sided die is rolled twice, and the sum of the two rolls is computed. What is the probability that the sum is a prime number?

A) $\frac{7}{18}$ B) $\frac{5}{12}$ C) $\frac{5}{11}$ D) $\frac{19}{36}$ E) NOTA

3. Given three distinct colors, in how many ways can the vertices of a square be distinctly colored? (Rotations are considered the same coloring.)

A) 18 B) 24 C) 27 D) 30 E) NOTA

4. There are 21 students in my calculus class. In how many ways can I form three teams of seven students each? Assume the three teams are enumerated 1, 2, 3 to denote a difference in the teams; thus, for example, seven people on team 1 is distinct from the same seven people being on team 2.

A) $\binom{23}{2}$ B) $3 \cdot 7!$ C) $3! \cdot \binom{21}{7}$ D) $\binom{21}{7}\binom{14}{7}$ E) NOTA

5. Compute the probability that I get exactly three heads in a row by flipping five fair coins.

A) $\frac{3}{32}$ B) $\frac{5}{32}$ C) $\frac{3}{16}$ D) $\frac{1}{4}$ E) NOTA

6. A real number k is chosen from the interval $[-5, 5]$. Compute the probability that the polynomial $f(x) = x^3 - 6x^2 + 9x + k$ has exactly three real roots.

A) $\frac{1}{5}$ B) $\frac{2}{5}$ C) $\frac{3}{5}$ D) $\frac{4}{5}$ E) NOTA

7. Five friends go to the Las Vegas Cinema-plex movie theater, which has eight theaters. On each of six screens, there is a different movie playing, but on the other two screens, the same seventh movie is playing. (So there are only seven distinct movies playing.) In how many ways can the five friends see a specific movie, not on a specific screen, assuming each of them will actually see a movie?

A) 2520 B) 32,768 C) 78,125 D) 180,075 E) NOTA

8. Compute the probability that a randomly chosen divisor of 15,015 is prime.

A) $\frac{5}{32}$ B) $\frac{1}{6}$ C) $\frac{3}{16}$ D) $\frac{1}{5}$ E) NOTA

9. Three boys and seven girls are to line up in a row. In how many ways can they line up with the restriction that no two boys stand next to each other?

A) 252 B) 336 C) 453,600 D) 1,693,440 E) NOTA

10. Three boys and seven girls are to line up in a row. In how many ways can they line up with the restriction that all the boys stand next to each other?

A) 30,240 B) 40,320 C) 241,920 D) 604,800 E) NOTA

11. On the new nationwide television hit show, *Mu Alpha Theta's Got Talent*, three judges have to vote publicly on three performers, René, Leonhard, and Isaac, listing their order of preference. In how many ways can the judges vote so that two of them agree in their order of preference, while the third judge differs?

A) 30 B) 60 C) 90 D) 180 E) NOTA

12. Consider the set of nonnegative integer solutions (x, y, z) to the equation $x + y + z = 20$. If a solution triple is chosen at random from this set, what is the probability that it is a solution with positive integer solutions?

A) $\frac{34}{57}$ B) $\frac{68}{95}$ C) $\frac{57}{77}$ D) $\frac{190}{253}$ E) NOTA

13. I want to make up a new word. The new word can use any of the 26 letters of the English alphabet; can be any length from 1 to 26 letters; must have letters that follow an alphabetic progression; and can have no repeated letter. For example, "aqz" would work, but "qaz" would not; "forty" would work, but "fifty" would not. How many such words can I devise?

A) $2^{26} - 1$ B) 2^{26} C) $2^{26} + 1$ D) 26! E) NOTA

14. Nikoli tells Ludwig that if he randomly selects a math tournament date this year, there is a $1/4$ probability that a musical event is scheduled the same day. Ludwig tells Nikoli that that if he randomly selects a musical event date this year, there is a $1/3$ probability that a math tournament is scheduled the same day. Then, on a calendar, they highlight all days on which there is a math tournament or a musical event. If they now pick any date that is highlighted, what is the probability that a musical event occurs on that day?

A) $\frac{1}{12}$ B) $\frac{3}{8}$ C) $\frac{1}{2}$ D) $\frac{7}{12}$ E) NOTA

15. The four girls and three boys of the sketch comedy class at Keyandpeele High School must select a cast of five of their members to perform a skit. At least two of these members must be girls. How many different possible casts could the class make?

A) 12 B) 15 C) 18 D) 21 E) NOTA

16. Blaise and Pierre play a game. They roll a fair six-sided die. If they roll a 5 or a 6, then Pierre wins. If they roll a 1, Blaise wins. Otherwise, they roll the die again. When one person wins, they stop, and do not play the game anymore. What is the probability that Pierre eventually wins?

A) $\frac{1}{6}$ B) $\frac{1}{3}$ C) $\frac{2}{5}$ D) $\frac{2}{3}$ E) NOTA

17. What is the maximum number of points of intersection between 10 circles and 10 lines where the circles and lines are distinct?

A) 225 B) 290 C) 335 D) 360 E) NOTA

18. Compute the probability that a four-digit positive integer will have some number of repeated digits.

A) $\frac{62}{125}$ B) $\frac{68}{125}$ C) $\frac{683}{1250}$ D) $\frac{4}{7}$ E) NOTA

19. How many five-digit positive integers are there such that each digit is no greater than the one before it, going left-to-right?

A) 1286 B) 1364 C) 2001 D) 3002 E) NOTA

20. A fair six-sided die is rolled three times. The first roll determines the hundreds digit, the second roll determines the tens digit, and the third roll determines the units digit of a three-digit number. What is the probability that the three-digit number formed is a perfect square?

A) $\frac{2}{81}$ B) $\frac{7}{216}$ C) $\frac{1}{27}$ D) $\frac{1}{24}$ E) NOTA

21. Two real positive numbers x and y are randomly chosen such that $x + y \leq 4$. Compute the probability that $x + y \leq 2$.

A) $\frac{1}{4}$ B) $\frac{\sqrt{2}}{4}$ C) $\frac{1}{2}$ D) $\frac{\sqrt{2}}{2}$ E) NOTA

22. Violet dips a $4 \times 4 \times 4$ cube into blueberry-blue paint. After the paint dries, she cuts the cube into 64 $1 \times 1 \times 1$ pieces. She then picks a unit cube at random from the unit cube pieces and rolls it. Compute the probability that the face that lands up is painted.

A) $\frac{1}{16}$ B) $\frac{5}{24}$ C) $\frac{15}{64}$ D) $\frac{1}{4}$ E) NOTA

23. Blaise and Pierre play a different game. For this game, each of them rolls a fair standard six-sided die. If they both roll the same thing, then Pierre wins, and the game stops. If Blaise rolls a higher number than Pierre, then Blaise wins, and the game stops. If Pierre rolls a higher number than Blaise, they play again. They play until one of them wins, then they stop, and do not play anymore. Compute the probability that Blaise wins this game.

A) $\frac{5}{12}$ B) $\frac{7}{12}$ C) $\frac{3}{4}$ D) $\frac{5}{6}$ E) NOTA

24. Let $7/(7n+9)$ be the probability that Zarek ties his shoes together correctly on day n. Assuming each day is an independent event, compute the probability that Zarek does not tie his shoes correctly for days 1 through 100.

A) $\frac{7}{709}$ B) $\frac{9}{709}$ C) $\frac{23}{709}$ D) $\frac{686}{709}$ E) NOTA

25. The nine members of the Clarke County, Nevada School Board convene every other Tuesday. There are four Democrats, three Republicans, and two Independents on the Board. Before each meeting begins, they shake hands. However, Republicans and Democrats do not shake hands! How many handshakes occur between Board members?

A) 22 B) 24 C) 30 D) 36 E) NOTA

26. Chauncey loves to play basketball. The probability of Chauncey making 963 out of 2018 free-throws is the same as the probability of making 964 out of 2018 free-throws, and each free throw attempt result is independent of any other attempt. The probability of him not making a free-throw out of one attempt is the reduced fraction p/q. Compute $p+q$.

A) 1973 B) 2983 C) 3945 D) 4036 E) NOTA

27. Dots are arranged in a rectangular grid of four rows and n columns. Each dot is colored either yellow or green. Call a coloring *rectangle-free* if no four dots of the same color form a rectangle with horizontal and vertical sides. What is the maximum value of n that allows a rectangle-free coloring?

A) 6 B) 8 C) 9 D) 10 E) NOTA

28. Jedidiah has a list of 100 people, in alphabetical order. Zechariah is the last person on the list; he knows the meaning of life, and nobody else does. Jedidiah knows that someone on his list knows the meaning of life, but he does not know who it is, so he starts asking people from his list, in random order. He never asks the same person twice, and he stops once he knows the meaning of life, asking no more people. Let p be the probability that the people Jedidiah asks are asked in alphabetical order. What is the integer closest to e/p, where e is the base of the natural logarithm?

A) 49 B) 50 C) 99 D) 100 E) NOTA

29. I finally washed my socks and now I will hang them out to dry on a straight clothesline. I have four pairs of socks and I will hang them side by side. The socks in each pair are identical but the pairs themselves have different colors. How many different colors patterns can be made if no sock is allowed to be next to its mate?

A) 216 B) 576 C) 720 D) 864 E) NOTA

30. Robert, Roberto, Roberta, and Bob decide to each make a square quilt, so four quilts total. Each quilt consists of nine 1 foot by 1 foot squares sewn together into a bigger 3 foot by 3 foot square quilt. Each 1×1 square can be either pink, purple, or turquoise. Bob can use only pink squares for his quilt, but everyone else can use any of the three colors. After the four quilts are made, they are sewn together into one giant 6 foot by 6 foot square quilt. Let Q be the number of color combinations of the big quilt that can

be made. (Note: the quilt may be rotated, so the same pattern in a different orientation is only counted once.) Compute the remainder when Q is divided by 10.

A) 1 or 2 B) 3 or 4 C) 5 or 6 D) 7 or 8 E) NOTA